Chapter 5 Trigonometric Functions: Right Triangle Approach

5.2 Trigometry of Right Triangles

① Trigometric Ratios

② Using a Calculator

③ Finding an Unknown Side

④ Evaluating an Expression

⑤ Solving a Right Triangle

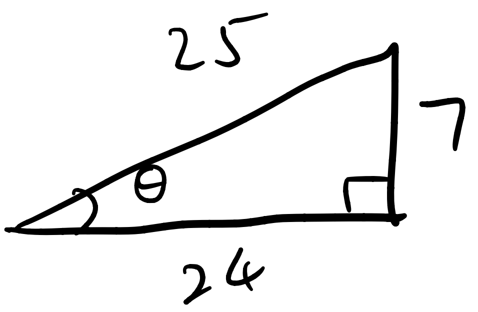
⑥ Applications

① Trigonometric Ratios

3. $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

$\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

4.



$\sin\theta = \frac{7}{25}$ $\csc\theta = \frac{25}{7}$

$\cos\theta = \frac{24}{25}$ $\sec\theta = \frac{25}{24}$

$\tan\theta = \frac{7}{24}$ $\cot\theta = \frac{24}{7}$

5.

$\sin\theta = \frac{40}{41}$

$40^2 + a^2 = 41^2$

$a = \sqrt{41^2 - 40^2}$

$= 9$

$\cos\theta = \frac{9}{41}$

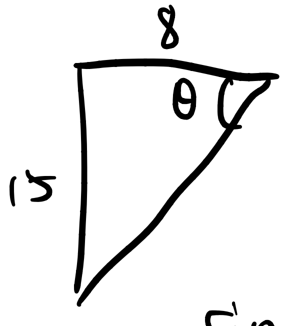
$\tan\theta = \frac{40}{9}$

$\csc\theta = \frac{41}{40}$

$\sec\theta = \frac{41}{9}$

$\cot\theta = \frac{9}{40}$

6.



$8^2 + 15^2 = h^2$

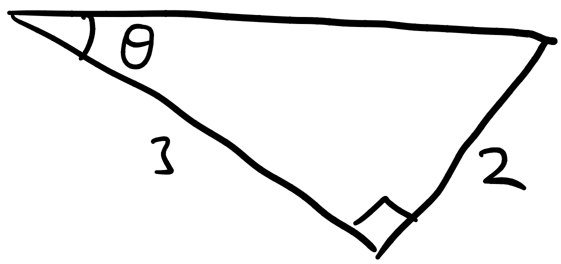
$h = 17$

$\sin\theta = \frac{15}{17}$ $\csc\theta = \frac{17}{15}$

$\cos\theta = \frac{8}{17}$ $\sec\theta = \frac{17}{8}$

$\tan\theta = \frac{15}{8}$ $\cot\theta = \frac{8}{15}$

7.



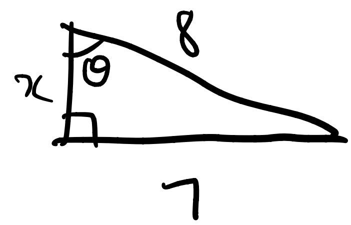
$h = \sqrt{3^2+2^2}$
$= \sqrt{13}$

$\sin\theta = \frac{2}{\sqrt{13}}$ $\csc\theta = \frac{\sqrt{13}}{2}$

$\cos\theta = \frac{3}{\sqrt{13}}$ $\sec\theta = \frac{\sqrt{13}}{3}$

$\tan\theta = \frac{2}{3}$ $\cot\theta = \frac{3}{2}$

8.



$x = \sqrt{8^2-7^2}$
$= \sqrt{15}$

$\sin\theta = \frac{7}{8}$ $\csc\theta = \frac{8}{7}$

$\cos\theta = \frac{\sqrt{15}}{8}$ $\sec\theta = \frac{8}{\sqrt{15}}$

$\tan\theta = \frac{7}{\sqrt{15}}$ $\cot = \frac{\sqrt{15}}{7}$

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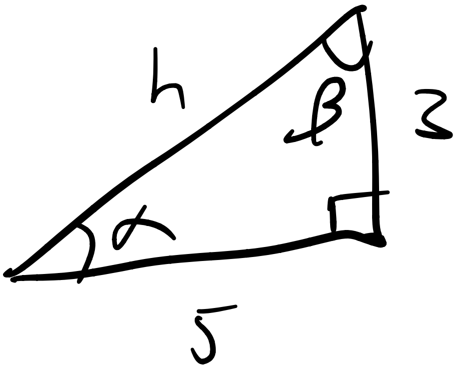
9.



$h = \sqrt{3^2 + 5^2}$
$= \sqrt{34}$

(a) $\sin\alpha = \dfrac{3}{\sqrt{34}}$

$\cos\beta = \dfrac{3}{\sqrt{34}}$

(b) $\tan\alpha = \dfrac{3}{5}$

$\cot\beta = \dfrac{3}{5}$

(c) $\sec\alpha = \dfrac{1}{\cos\alpha}$
$= \dfrac{\sqrt{34}}{5}$

$\csc\beta = \dfrac{\sqrt{34}}{5}$

② Using a Calculator

11. (a) $\sin 22°$, (b) $\cot 23°$

$\sin 22°$
$= 0.37461$

$\cot 23°$
$= 2.35585$

12. (a) $\cos 37°$
$= 0.79864$

(b) $\csc 48°$
$= 1.34563$

13. (a) $\sec 13°$
$= 1.02630$

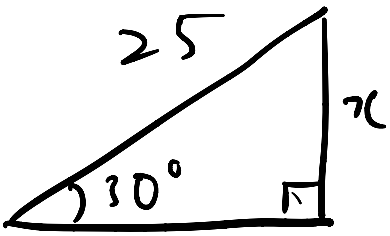
(b) $\tan 51°$
$= 1.23490$

14. (a) $\csc 10°$
$= 5.75877$

(b) $\sin 46°$
$= 0.71934$

③ Finding an Unknown Side

15.



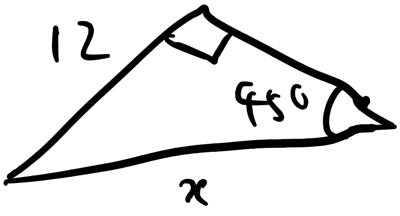
$$\sin 30^\circ = \frac{x}{25} = \frac{1}{2}$$

$$x = \frac{25}{2}$$

2

$$x = 25 \sin 30^\circ$$
$$= 25\left(\frac{1}{2}\right)$$
$$= \frac{25}{2}$$

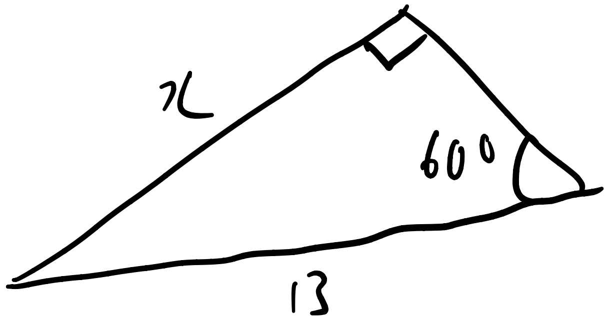
16.



$$\sin 45^\circ = \frac{12}{x} = \frac{1}{\sqrt{2}}$$

$$x = 12\sqrt{2}$$

17.

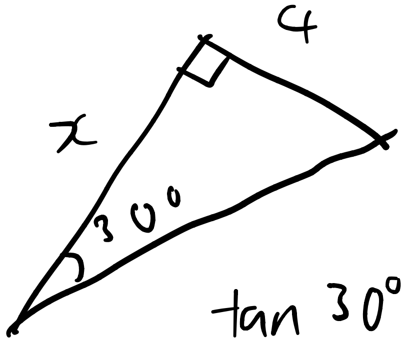


$$\sin 60° = \frac{x}{13} = \frac{\sqrt{3}}{2}$$

$$x = \frac{13\sqrt{3}}{2}$$

$$= 11.25833$$

18.



$$\tan 30° = \frac{1}{\sqrt{3}}$$

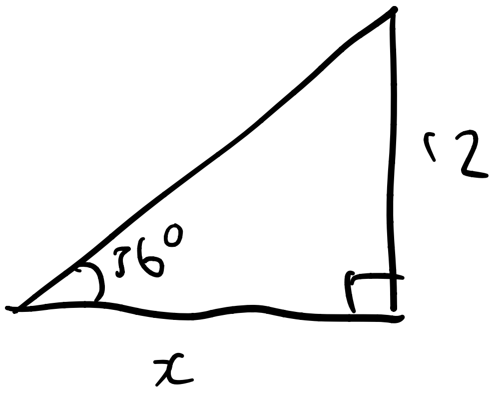
$$\tan 30° = \frac{4}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{4}{x}$$

$$x = 4\sqrt{3}$$

$$= 6.92820$$

19.

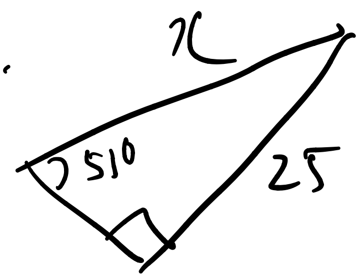


$$\tan 36° = \frac{12}{x}$$

$$x = \frac{12}{\tan 36°}$$

$$= 16.51658$$

20.

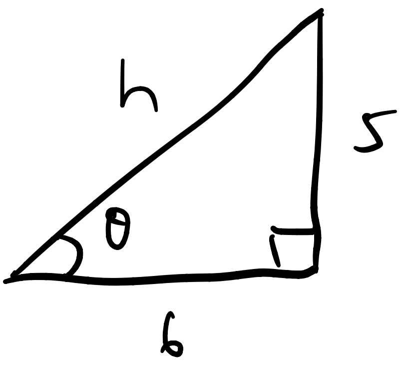


$$\sin 53° = \frac{25}{x}$$

$$x = \frac{25}{\sin 53°}$$

$$= 31.3$$

23. $\tan\theta = \frac{5}{6}$



$5^2 + 6^2 = h^2$

$h = \sqrt{25+36}$

$= \sqrt{61}$

$\sin\theta = \frac{5\sqrt{61}}{61}$

$\cos\theta = \frac{6\sqrt{61}}{61}$

$\csc\theta = \frac{\sqrt{61}}{5}$

$\sec\theta = \frac{\sqrt{61}}{6}$

$\cot\theta = \frac{6}{5}$

29. $\sin\frac{\pi}{6} + \cos\frac{\pi}{6}$ (4) Evaluating an Expression

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1+\sqrt{3}}{2}$$

30. $\sin 30° \csc 30°$

$$= \frac{1}{2}(2)$$

$$= 1$$

31. $\sin 30° \cos 60° + \sin 60° \cos 30°$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= 1$$

32. $(\sin 60°)^2 + (\cos 60°)^2 = 1$

$$(\sin 60°)^2 + (\cos 60°)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

33. $(\cos 30^\circ)^2 - (\sin 30^\circ)^2$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{1}{2}$$

34. $$\left(\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3}\right)^2$$

$$= \left(\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)\right)^2$$

$$= \left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)^2$$

$$= \frac{(\sqrt{3}-1)^2}{(2\sqrt{2})^2}$$

$$= \frac{3 - 2\sqrt{3} + 1}{8}$$

$$= \frac{4 - 2\sqrt{3}}{8}$$

$$= \frac{2-\sqrt{3}}{4}$$

35. $\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{6}\right)^2$

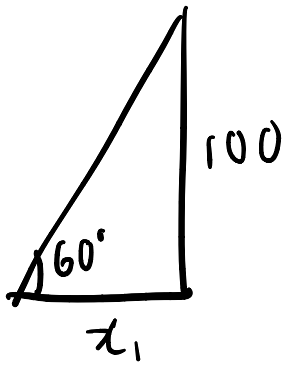
$$=\left(\frac{\sqrt{2}}{2}+\frac{1}{2}\right)^2$$

$$=\left(\frac{\sqrt{2}+1}{2}\right)^2$$

$$=\frac{2+2\sqrt{2}+1}{4}$$

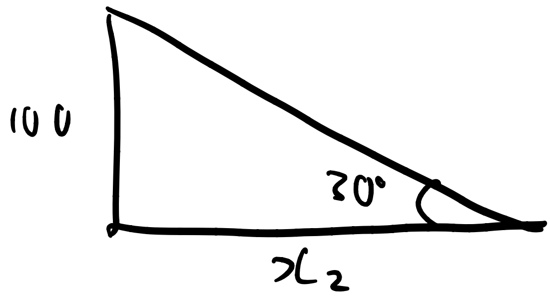
$$=\frac{3}{4}+\frac{\sqrt{2}}{2}$$

47.



$$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1} = \frac{100}{x_1}$$

$$x_1 = \frac{100}{\sqrt{3}}$$



$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{100}{x_2}$$

$$x_2 = 100\sqrt{3}$$

$\boxed{12}$

$$\tan\frac{\pi}{3} = \frac{100}{x_1} \qquad \tan\frac{\pi}{6} = \frac{100}{x_2}$$

$$x_1 = \frac{100}{\tan\frac{\pi}{3}} \qquad x_2 = \frac{100}{\tan\frac{\pi}{6}}$$

$$x = x_1 + x_2$$

$$= \frac{100}{\tan\frac{\pi}{3}} + \frac{100}{\tan\frac{\pi}{6}}$$

$$x = x_1 + x_2$$

$$= \frac{100}{\sqrt{3}} + 100\sqrt{3}$$

$$= \frac{100}{\sqrt{3}} + 100\frac{\sqrt{3}}{\sqrt{3}}(\sqrt{3})$$

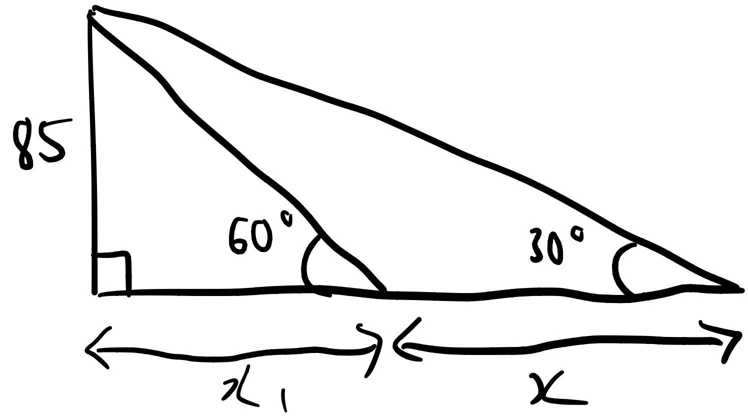
$$= \frac{100}{\sqrt{3}} + \frac{300}{\sqrt{3}}$$

$$= \frac{400}{\sqrt{3}}$$

$$= \frac{400\sqrt{3}}{3}$$

$$= 230.94°$$

48.



$$\tan 60^\circ = \sqrt{3} = \frac{85}{x_1}$$

$$x_1 = \frac{85}{\sqrt{3}}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{85}{x_2}$$

$$x_2 = 85\sqrt{3}$$

$$x_2 = x_1 + x$$

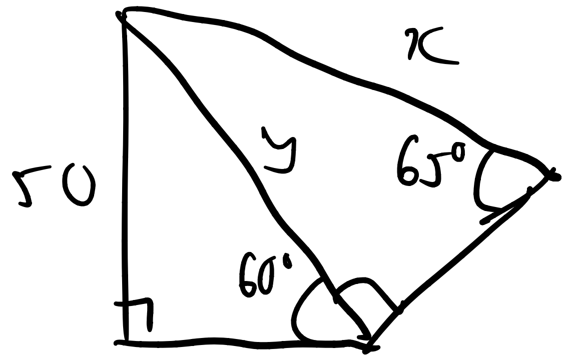
$$x = x_2 - x_1$$

$$= 85\sqrt{3} - \frac{85}{\sqrt{3}}$$

$$= \frac{85\sqrt{3}\,(\sqrt{3})}{\sqrt{3}} - \frac{85}{\sqrt{3}}$$

$$= \frac{170}{\sqrt{3}} \text{ or } 98.1$$

49.



$$\sin 60° = \frac{50}{y}$$

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\frac{50}{y} = \frac{\sqrt{3}}{2}$$

$$y = \frac{100}{\sqrt{3}}$$

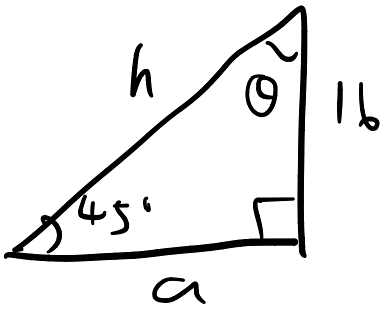
$$\sin 65° = \frac{\frac{100}{\sqrt{3}}}{x}$$

$$x = \frac{\frac{100}{\sqrt{3}}}{\sin 65°}$$

$$= 63.7$$

(5) Solving a Right Triangle

37.



$\tan 45^\circ = \dfrac{16}{a}$

$a = \dfrac{16}{\tan 45^\circ}$

$= \dfrac{16}{1}$

$= 16$

$\sin 45^\circ = \dfrac{16}{h}$

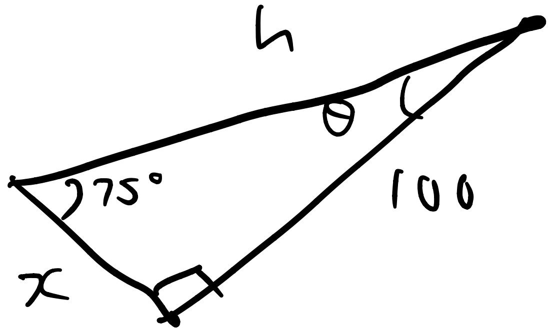
$h = \dfrac{16}{\sin 45^\circ}$

$= \dfrac{16}{\frac{1}{\sqrt{2}}}$

$= 16\sqrt{2}$

$\theta = 45^\circ$

38.



$\theta = 90 - 75$

$= 15^\circ$

$\sin 75^\circ = \dfrac{100}{h}$

$h = \dfrac{100}{\sin 75^\circ}$

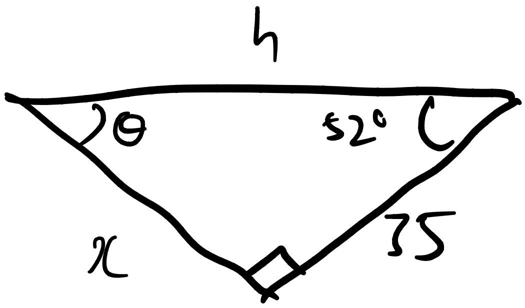
$= 103.53$

$\tan 75^\circ = \dfrac{100}{x}$

$x = \dfrac{100}{\tan 75^\circ}$

$= 26.79$

39.



$\theta = 90 - 52$

$= 38°$

$\tan 52° = \dfrac{x}{35}$

$x = 35 \tan 52°$

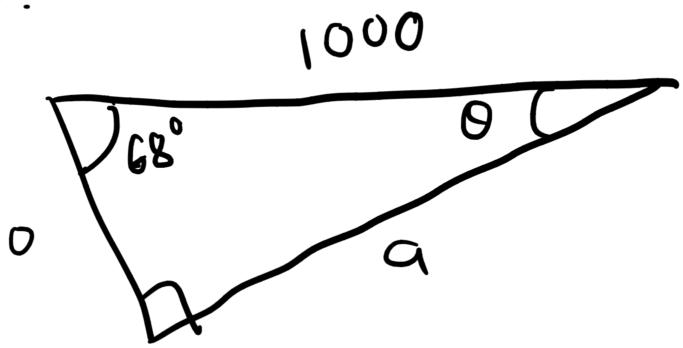
$= 44.80$

$\cos 52° = \dfrac{35}{h}$

$h = \dfrac{35}{\cos 52°}$

$= 56.8$

40.



$\theta = 180° - 68° - 90°$

$= 22°$

$\sin 22° = \dfrac{o}{100}$

$o = 1000 \sin 22°$

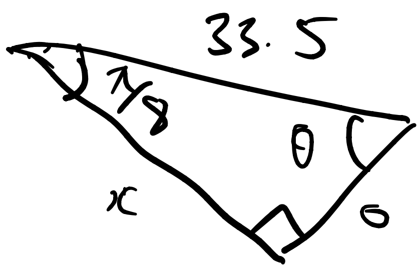
$= 374.61$

$\cos 22° = \dfrac{a}{100}$

$a = 1000 \cos 22°$

$= 927.18$

41.



$\theta = \dfrac{\pi}{2} - \dfrac{\pi}{8}$

$= \dfrac{3\pi}{8}$

$\sin \dfrac{\pi}{8} = \dfrac{o}{33.5}$

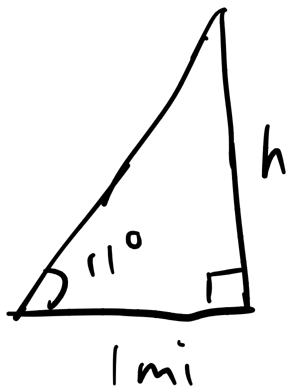
$o = 12.82$

$\cos \dfrac{\pi}{8} = \dfrac{x}{33.5}$

$x = 30.95$

53.

(6) Applications



$\tan 11° = 0.19438$

$\tan 11° = \frac{h}{1}$

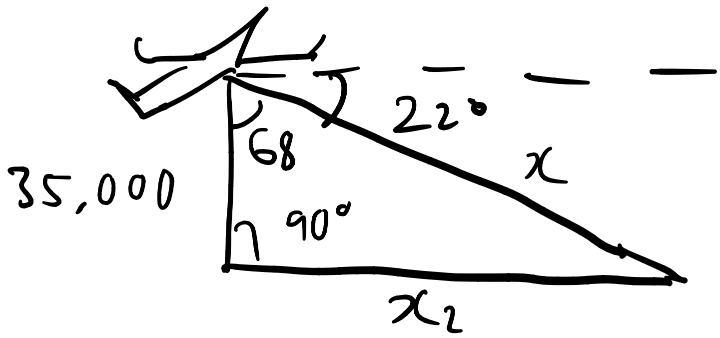
$h = 0.19438$ mi

$= 0.19438 \times 1609.344$ m

$= 312.82$ m

$\boxed{2}$ ~1026 ft

54.



(a) $$\cos 68° = \frac{35000}{x}$$

$$x = \frac{35000}{\cos 68°}$$

$$= 93451.4 \text{ ft}$$

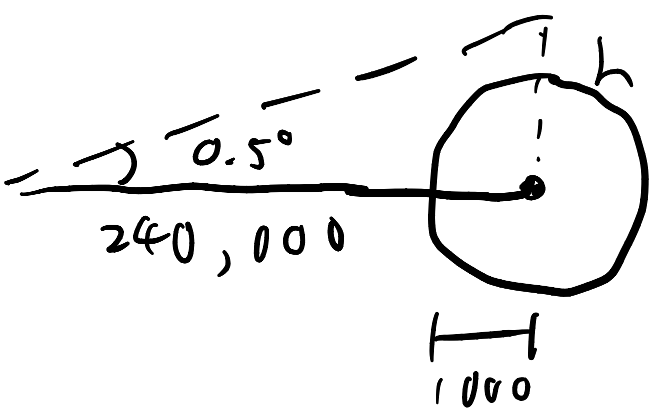
away

(b) $$\tan 68° = \frac{x_2}{35000}$$

$$x_2 = 35000 \tan 68°$$

$$= 86628 \text{ ft}$$

55.

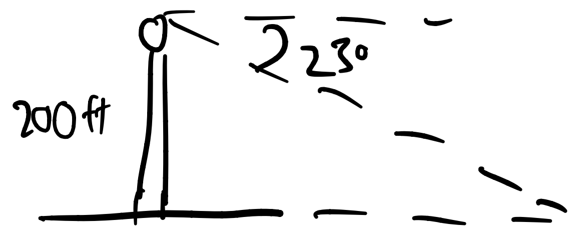


(a) $\tan 0.5 = \dfrac{h}{240000}$

$$h = 240000 \tan 0.5^\circ$$
$$= 2094.4 \text{ mi}$$

(b) No, because the beam diverges 2094 mi away from the center of the moon which is more than its radius

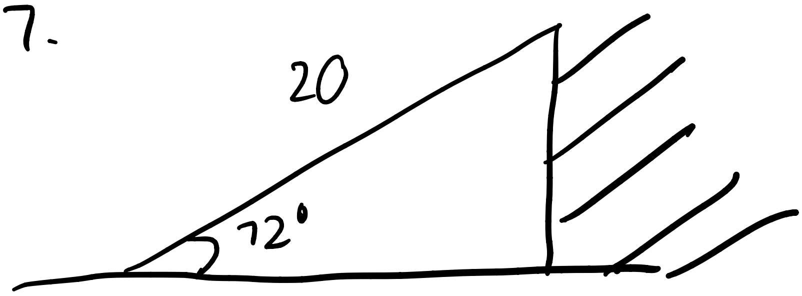
56.



$$\tan 23^\circ = \frac{200}{x}$$

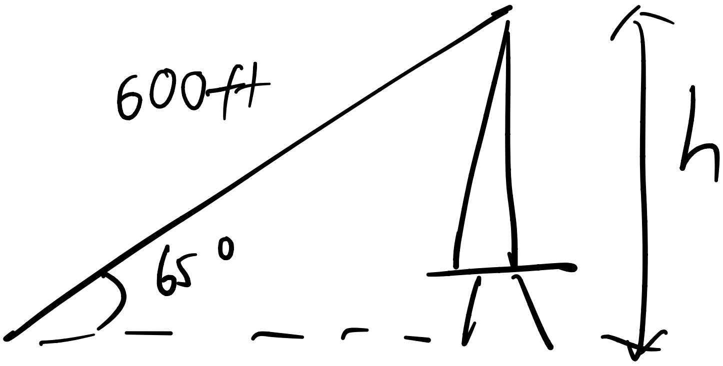
$$x = \frac{200}{\tan 23^\circ}$$
$$= 471.2 \text{ ft}$$

57.



$h = 20 \sin 72°$
$= 19.02$ ft

58.



$h = 600 \sin 65°$

$= 543.78$ ft

5.3 Trigometric Functions of Angles

① Reference Angle
② Values of Trigonometric Functions
③ Expressing One Trigonometric Function in Terms of Another
④ Values of Trigonometric Functions II
⑤ Area of a Triangle
⑥ Applications

① Reference Angle

5. (a) 120° (b) 200° (c) 285°

$\bar{\theta} = 60^\circ$ $\bar{\theta} = 20^\circ$ $\bar{\theta} = 75^\circ$

6. (a) 175° (b) 310° (c) 730°

$\bar{\theta} = 5^\circ$

$\bar{\theta} = 360 - 310^\circ$
$= 50^\circ$

$= 730^\circ - 2(360^\circ)$
$= 10^\circ$

7. (a) 225° (b) 810° (c) -105°

$\bar{\theta} = 225^\circ - 180^\circ$
$= 45$

$\bar{\theta} = 810^\circ - 2(360^\circ)$
$= 90^\circ$

$\bar{\theta} = -105 + 180$
$= 75^\circ$

8. (a) 99° (b) -199° (c) 359°

$= 180^\circ - 99^\circ$
$= 81^\circ$

$360 - 199$
$= 161$
$180^\circ - 161^\circ$
$= 19^\circ$

$360^\circ - 359^\circ$
$= 1^\circ$

9. (a) $\frac{7\pi}{10}$ (b) $\frac{9\pi}{8}$ (c) $\frac{10}{3}\pi$

$\pi - \frac{7}{10}\pi$
$= \frac{3}{10}\pi$

$\frac{9\pi}{8} - \pi$
$= \frac{1}{8}\pi$

$\frac{10}{3}\pi - 3\pi$
$= \frac{1}{3}\pi$

② $\frac{}{6}$ Values of Trigonometric Functions

13. $\cos 150°$

$= -\cos 30°$

$= -\frac{\sqrt{3}}{2}$

16. $\sin(-30°)$

$= -\sin 30°$

$= -\frac{1}{2}$

14. $\sin 240°$

$\bar{\theta}° = 240 - 180$

$= 60°$

$\sin 240° = -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

15. $\tan 330°$

$\bar{\theta} = 360 - 330$

$= 30°$

$\tan 330° = -\tan 30°$

$= -\frac{1}{\sqrt{3}}$

16. $\sin(-30^\circ)$

$= -\sin 30^\circ$

$= -\frac{1}{2}$

17. $\cot(-120^\circ)$

$= \frac{1}{\tan(-120^\circ)}$

$= \frac{1}{-\tan 120^\circ}$

$= \frac{1}{-(-\tan 60^\circ)}$

$= \frac{1}{\tan 60^\circ}$

$= \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{3}$

37. $\sin\theta < 0$ and $\cos\theta < 0$

Quadrant 3

③ Expressing One Trigonometric Function in Terms of Another

41. $\tan\theta$, $\cos\theta$, θ in Quadrant III

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{-\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

42. $\cot\theta$, $\sin\theta$ in Quadrant II

$$\cot\theta = \pm\frac{1}{\sin\theta}\sqrt{1-\sin^2\theta}$$

$\because$ Quadrant II

$$\cot\theta = -\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$$

$$1+\cot^2\theta = \csc^2\theta$$

$$\cot\theta = \pm\sqrt{\csc^2\theta - 1}$$

$$\cot\theta = \pm\sqrt{\frac{1}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}}$$

$$= \pm\frac{1}{\sin\theta}\sqrt{1-\sin^2\theta}$$

43. $\cos\theta$, $\sin\theta$ θ in Quadrant IV

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\cos\theta^2 = 1-\sin^2\theta$$

$$\cos\theta = +\sqrt{1-\sin^2\theta}$$

(Q4)

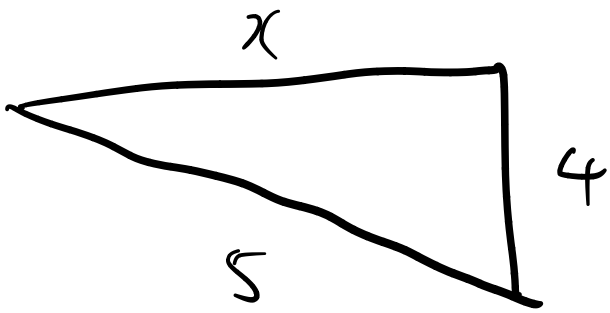
44. $\sec\theta$, $\sin\theta$ θ in Quadrant I

$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{1}{\sqrt{1-\sin^2\theta}}$$

4/6 Values of Trigonometric Functions II

47. $\sin\theta = -\frac{4}{5}$, θ in Quadrant 4



$$x^2 + 4^2 = 5^2$$

$$x = \sqrt{5^2 - 4^2}$$

$$= 3$$

$$\cos\theta = \frac{3}{5}$$

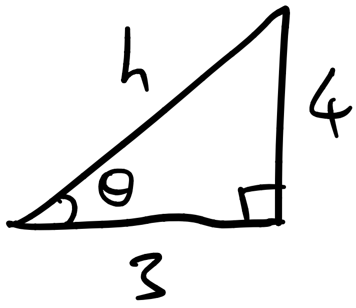
$$\tan\theta = -\frac{4}{3}$$

$$\csc\theta = -\frac{5}{4}$$

$$\sec\theta = \frac{5}{3}$$

$$\cot = -\frac{3}{4}$$

48. $\tan\theta = \frac{4}{3}$, Quadrant III



$h = \sqrt{4^2+3^2}$

$= 5$

$\sin\theta = -\frac{4}{5}$

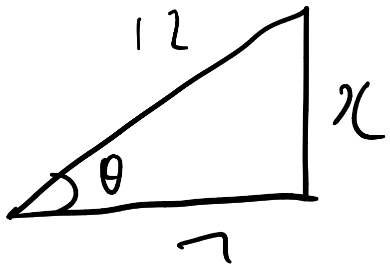
$\cos\theta = -\frac{3}{5}$

$\csc\theta = -\frac{5}{4}$

$\sec\theta = -\frac{5}{3}$

$\cot\theta = \frac{3}{4}$

49. $\cos\theta = \frac{7}{12}$, $\sin\theta < 0$



$$7^2 + x^2 = 12^2$$
$$x = \sqrt{12^2 - 7^2}$$
$$= \sqrt{95}$$

$$\sin\theta = -\frac{\sqrt{95}}{12}$$

$$\tan\theta = -\frac{\sqrt{95}}{7}$$

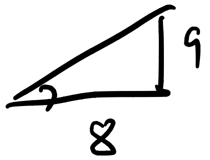
$$\csc\theta = -\frac{12}{\sqrt{95}}$$

$$\sec\theta = \frac{12}{7}$$

$$\cot\theta = -\frac{7}{\sqrt{95}}$$

50. $\cot\theta = -\frac{8}{9}$, $\cos\theta > 0$

$$\tan\theta = -\frac{9}{8}$$



$$h = \sqrt{9^2 + 8^2}$$
$$= \sqrt{145}$$

$$\csc\theta = -\frac{\sqrt{145}}{9}$$

$$\sec\theta = \frac{\sqrt{145}}{8}$$

$$\sin\theta = -\frac{9}{\sqrt{145}}$$

$$\cos\theta = \frac{8}{\sqrt{145}}$$

51. $\csc\theta = 2$, θ in Quadrant I

$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\frac{1}{2}$$

$$= 30^\circ$$

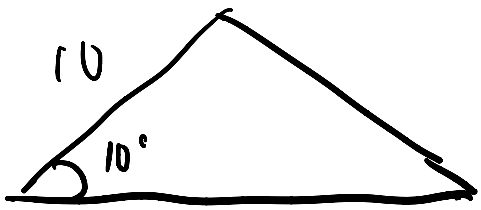
⑤ Area of a Triangle

57. $\frac{1}{2}ab\sin\theta$

$= \frac{1}{2}(9)(7\sin 72^\circ)$

$= 29.96$

58.

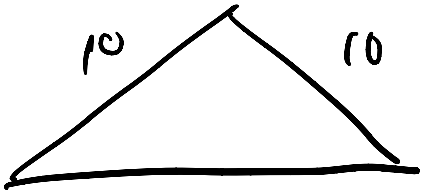


$A = \frac{1}{2}ab\sin\theta$

$= \frac{1}{2}22(10\sin 10^\circ)$

$= 19.10$

59.



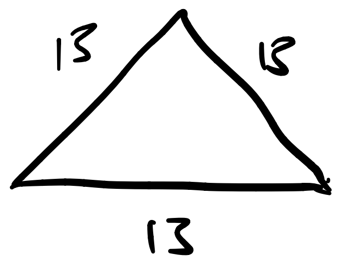
$\theta = \frac{180}{3}$
$= 60^\circ$

$A = \frac{1}{2}ab\sin\theta$

$= \frac{1}{2}10(10\sin 60^\circ)$

$= 5(10\frac{\sqrt{3}}{2}) = 25\sqrt{3}$

60.



$$A = \frac{1}{2} ab\sin\theta$$

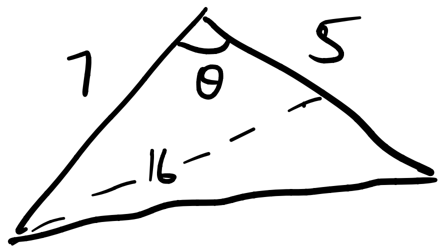
$$= \frac{1}{2}(13)(13\sin 60^\circ)$$

$$= \frac{1}{2}(13)(13)\left(\frac{\sqrt{3}}{2}\right)$$

$$= 73.179$$

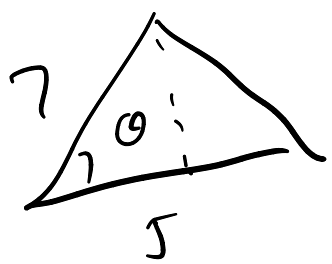
$$= 73.18$$

61.



$$A = \frac{1}{2} ab\sin\theta$$

$$16 = \frac{1}{2} 7(5\sin\theta)$$



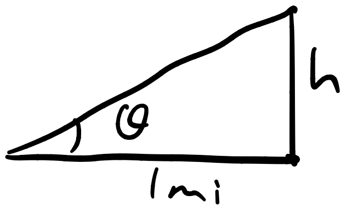
$$\sin\theta = \frac{32}{35}$$

$$\theta = \sin^{-1}\frac{32}{35}$$

$$= 66.10^\circ$$

6/6 Applications

65.



(a)

$$\tan\theta = \frac{h}{1\text{mi}}$$

$$h = \tan\theta\,(1\text{mi})$$

$$= 5280\tan\theta \text{ ft}$$

(b)

θ	$20°$	$60°$	$80°$	$85°$
h	1922	9145	29,944	60,351

66.

Area of square $= 10 \times 10\sin\theta = 100\sin\theta$

Area of each triangle $= h \times l \times \frac{1}{2}$

$= 10\sin\theta \times 10\cos\theta \times \frac{1}{2}$

$= 50\sin\theta\cos\theta$

Total Area $A(\theta) =$ square $+$ 2 triangles

$$= 100\sin\theta + 2(50\sin\theta\cos\theta)$$

$$= 100\sin\theta + 100\sin\theta\cos\theta$$

$\therefore$ shown

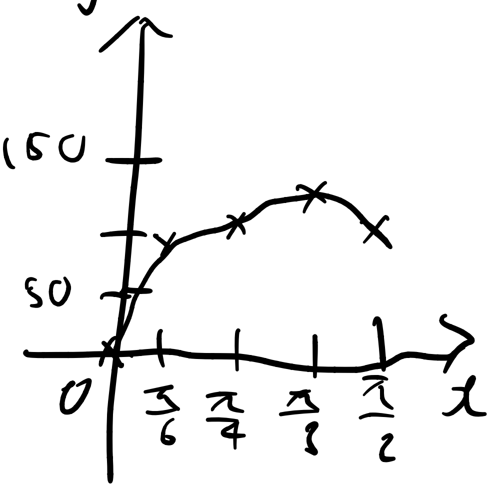
(b)

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	π
A	0	93.3	120.71	129.90	100

$$A\left(\frac{\pi}{6}\right) = 100\left(\frac{1}{2}\right) + 100\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= 50 + 25\sqrt{3}$$

$$= 93.3$$



$$A\left(\frac{\pi}{4}\right) = 100\left(\frac{1}{\sqrt{2}}\right) + 100\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{100\sqrt{2}}{2} + \frac{100}{2}$$

$$= 50\sqrt{2} + 50$$

$$= 120.71$$

$$A\left(\frac{\pi}{3}\right) = 100\left(\frac{\sqrt{3}}{2}\right) + 100\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{200\sqrt{3}}{4} + \frac{100\sqrt{3}}{4}$$

$$= \frac{300\sqrt{3}}{4}$$

$$= 129.90$$

$$A\left(\frac{\pi}{2}\right) = 100(1) + 0$$

$$= 100$$

(c) $\theta \approx \frac{\pi}{3}$

$\boxed{2}$ from graph, largest area is achieved when $\theta \approx 1.047$ rad $\approx 60°$

67. $d = 20\sin\theta$

$w = 20\cos\theta$

(a) $A(\theta) = 20\sin\theta \times 20\cos\theta$

$= 400\sin\theta\cos\theta$

(b) $A\left(\frac{\pi}{6}\right) = 20\left(\frac{1}{2}\right) \times 20\left(\frac{\sqrt{3}}{2}\right)$

$= 100\sqrt{3}$

$A\left(\frac{\pi}{4}\right) = 20\left(\frac{1}{\sqrt{2}}\right) \times 20\left(\frac{1}{\sqrt{2}}\right)$

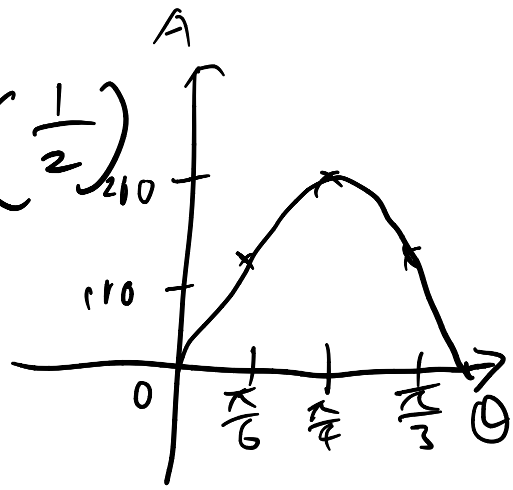
$= 200$

$A\left(\frac{\pi}{3}\right) = 20\left(\frac{\sqrt{3}}{2}\right) \times 20\left(\frac{1}{2}\right)$

$= 100\sqrt{3}$

$A\left(\frac{\pi}{2}\right) = 20 \times 0$

$= 0$



(c) $d = 20 \sin\left(\frac{\pi}{4}\right)$

$= 20\left(\frac{1}{\sqrt{2}}\right)$

$= 10\sqrt{2}$

$w = 20 \sin\left(\frac{\pi}{4}\right)$

$= 10\sqrt{2}$

68. Strength of a beam $\propto$ width and square of depth

width $= 20\cos\theta$

depth $= 20\sin\theta$

$\therefore$ Strength $= k(20\cos\theta)(20\sin\theta)^2$

$= 8000k\cos\theta\sin^2\theta$

69. $R = \dfrac{V_0^2 \sin(2\theta)}{g}$, $H = \dfrac{V_0^2 \sin^2\theta}{2g}$

(a) $R = \dfrac{12^2 \sin\left(2\left(\frac{\pi}{6}\right)\right)}{32}$ 3.897 ft

$= \dfrac{144\left(\frac{\sqrt{3}}{2}\right)}{32}$

$= \dfrac{9}{2}\left(\frac{\sqrt{3}}{2}\right) = \dfrac{9\sqrt{3}}{4}$

$H = \dfrac{12^2\left(\sin\frac{\pi}{6}\right)^2}{2(32)}$ 0.5625 ft

$= \dfrac{144\left(\frac{1}{2}\right)^2}{2(32)}$

$= \dfrac{18}{32} = \dfrac{9}{16}$

(b) $R = \dfrac{12^2 \sin\left(2\left(\frac{\pi}{6}\right)\right)}{5.2}$ $\qquad H = \dfrac{144\left(\frac{1}{2}\right)^2}{2(5.2)}$

$= 13.84615$ $\qquad = 3.46$ ft

solution says 23.982 ft

5.4 Inverse Trigonometric Functions and Right Triangles

① Evaluating Inverse Trigonometric Functions

② Evaluating Inverse Trigonometric Functions II

7 Sub-headers

① Evaluating Inverse Trigonometric Functions

5. (a) $\sin^{-1} 1$

$= \frac{\pi}{2}$ rad

(b) $\cos^{-1} 0$

$= \frac{\pi}{2}$ rad

(c) $\tan^{-1}\sqrt{3}$

$= \frac{\pi}{3}$ rad

6. (a) $\sin^{-1} 0$

$= 0$ rad

(b) $\cos^{-1}(-1)$

$= \pi$ rad

(c) $\tan^{-1} 0$

$= 0$ rad

7. (a) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\because$ only defined between $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$= -\frac{\pi}{4}$ rad

(b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $[0, \pi]$

$= \frac{3\pi}{4}$ rad

(c) $\tan^{-1}(-1)$

$= -\frac{\pi}{4}$ rad

8. (a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= -\frac{\pi}{3}$ rad

(b) $\cos^{-1}\left(-\frac{1}{2}\right)$

$= \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$ rad

(c) $\tan^{-1}(-\sqrt{3})$

$= -\tan^{-1}(\sqrt{3})$

$= -\frac{\pi}{3}$ rad

② Evaluating Inverse Trigonometric Functions II

9. $\sin^{-1}(0.30)$

$= 0.30469$

10. $\cos^{-1}(-0.2)$

$= \pi - \cos^{-1}(0.2)$

$= 1.77215$

11. $\cos^{-1}\frac{1}{3}$

$= 1.23096$

12. $\sin^{-1}\frac{5}{6}$

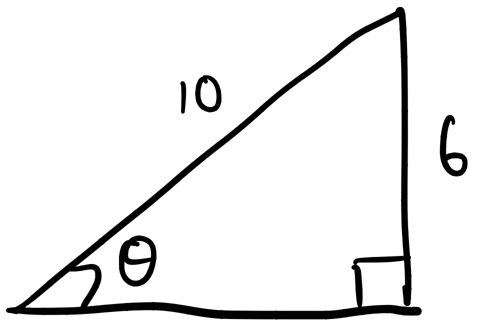
$= 0.98511$

13. $\tan^{-1} 3$

$= 1.24905$

③ Finding Angles in Right Triangles

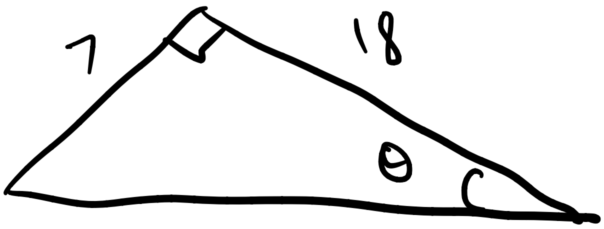
17.



$$\sin\theta = \frac{6}{10}$$
$$= \frac{3}{5}$$

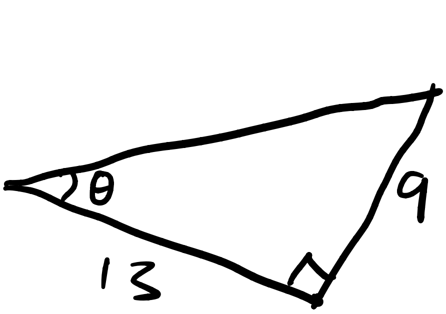
$$\sin^{-1}\frac{3}{5} = 36.9°$$

18.



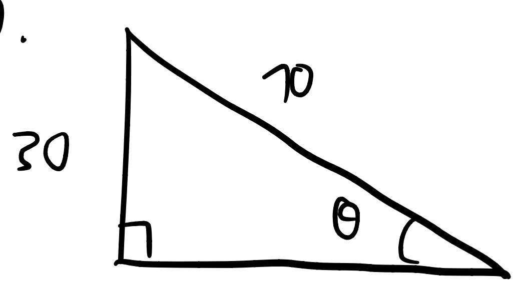
$$\tan\theta = \frac{7}{18}$$
$$\theta = \tan^{-1}\frac{7}{18}$$
$$= 21.3°$$

19.



$$\tan\theta = \frac{9}{13}$$
$$\theta = \tan^{-1}\frac{9}{13}$$
$$= 34.7°$$

20.

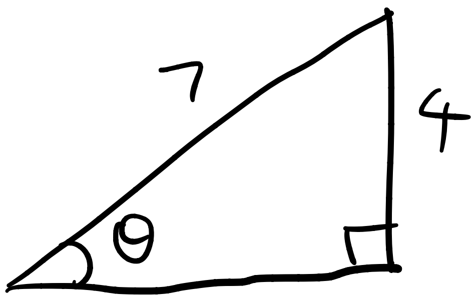


$$\sin\theta = \frac{30}{70}$$

$$\theta = \sin^{-1}\frac{3}{7}$$

$$= 25.4^\circ$$

21.

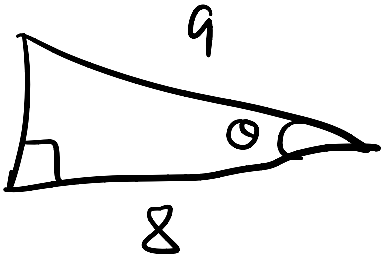


$$\sin\theta = \frac{4}{7}$$

$$\theta = \sin^{-1}\frac{4}{7}$$

$$= 34.8^\circ$$

22.



$$\cos\theta = \frac{8}{9}$$

$$\theta = \cos^{-1}\frac{8}{9}$$

$$= 27.3^\circ$$

4 Basic Trigonometric Equations

23. $\sin\theta = \frac{2}{3}$

$\theta = \sin^{-1}\frac{2}{3}$

$= 41.8°$

$180 - 41.8 = 138.2°$

$\therefore \theta = 41.8°, 138.2°$

24. $\cos\theta = \frac{3}{4}$

$\theta = \cos^{-1}\frac{3}{4}$

$= 41.4°$

25. $\cos\theta = -\frac{2}{5}$

$\theta = 180 - \cos^{-1}\frac{2}{5}$

$= 113.6°$

26. $\tan\theta = -20$

$\theta = -\tan^{-1}20$

$= -87.1^\circ$

$\theta = -87.1 + 180$

$= 92.9^\circ$

27. $\tan\theta = 5$

$\theta = \tan^{-1}5$

$= 78.7^\circ$

28. $\sin\theta = \frac{4}{5}$

$\theta = \sin^{-1}\frac{4}{5}$

$= 53.1^\circ$

$180^\circ - 53.1^\circ = 126.9^\circ$

$\therefore \theta = 53.1^\circ, 126.9^\circ$

(5) Value of an expression

29. $\cos\left(\sin^{-1}\frac{4}{5}\right)$

$$= \sqrt{1 - \sin^2\left(\sin^{-1}\frac{4}{5}\right)}$$

$$= \sqrt{1 - \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{25}{25} - \frac{16}{25}}$$

$$= \frac{3}{5}$$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$\because \cos\theta$ is positive between $[-\pi/2, \pi/2]$

30. $\cos\left(\tan^{-1}\frac{4}{3}\right)$

$$= \frac{1}{\sqrt{\tan^2\left(\tan^{-1}\frac{4}{3}\right) + 1}}$$

$$= \frac{1}{\sqrt{\left(\frac{4}{3}\right)^2 + \frac{9}{9}}}$$

$$= \frac{1}{\frac{5}{3}}$$

$$= \frac{3}{5}$$

$\tan^2\theta + 1 = \sec^2\theta$

$\frac{1}{\cos^2\theta} = \tan^2\theta + 1$

$\cos^2\theta = \frac{1}{\tan^2\theta + 1}$

$\cos\theta = +\sqrt{\frac{1}{\tan^2\theta + 1}}$

($\tan^{-1}$ is defined between $[-\pi/2, \pi/2]$ which $\cos\theta$ is +ve)

31. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-\sin^2\theta}}$$

Let $u = \sin^{-1}\frac{12}{13}$,

$$\sec u = \frac{1}{\sqrt{1-\sin^2 u}}$$
$$= \frac{1}{\sqrt{1-\left(\sin\left(\sin^{-1}\frac{12}{13}\right)\right)^2}}$$
$$= \frac{1}{\sqrt{1-\left(\frac{12}{13}\right)^2}}$$
$$= \frac{1}{\frac{5}{13}}$$
$$= \frac{13}{5}$$

32. $\csc\left(\cos^{-1}\frac{7}{25}\right)$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\sqrt{1-\cos^2\theta}}$$

Let $\cos^{-1}\frac{7}{25} = u$,

$$\csc(u) = \frac{1}{\sqrt{1-\cos^2 u}}$$
$$= \frac{1}{\sqrt{1-\left(\cos\left(\cos^{-1}\frac{7}{25}\right)\right)^2}} = \frac{25}{24}$$

⑥ Algebraic Expressions

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$\because \sin^{-1}\theta$ only defined between $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\therefore \cos\theta = \sqrt{1-\sin^2\theta}$

35. $\cos(\sin^{-1}x)$

$= \sqrt{1-\sin^2(\sin^{-1}x)}$

$= \sqrt{1-(x)^2}$

$= \sqrt{(1+x)(1-x)}$

36. $\sin(\tan^{-1}x)$

Let $u = \tan^{-1}x$,

$1+\cot^2 u = \csc^2 u$

$\frac{1}{\sin^2 u} = 1 + \frac{1}{\tan^2 u}$

$= \frac{\tan^2 u + 1}{\tan^2 u}$

$\sin^2 u = \frac{\tan^2 u}{\tan^2 u + 1}$

$\sin u = \pm \frac{\tan u}{\sqrt{\tan^2 u + 1}}$

$\sin(\tan^{-1}x)$

$= \pm \frac{\tan(\tan^{-1}x)}{\sqrt{\tan^2(\tan^{-1}x)+1}}$

$= \pm \frac{x}{\sqrt{x^2+1}}$

$\boxed{\text{B}}$ $\sin(\tan^{-1}x) = \frac{x}{\sqrt{x^2+1}}$

37. $\tan(\sin^{-1}x)$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

Let $\sin^{-1}x = u$,

$$\tan u = \frac{\sin u}{\sqrt{1-\sin^2 u}}$$

$$= \frac{\sin(\sin^{-1}x)}{\sqrt{1-(\sin(\sin^{-1}x))^2}}$$

$$= \frac{x}{\sqrt{1-x^2}}$$

38. $\cos(\tan^{-1}x)$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cos^2\theta = \frac{1}{\tan^2\theta + 1}$$

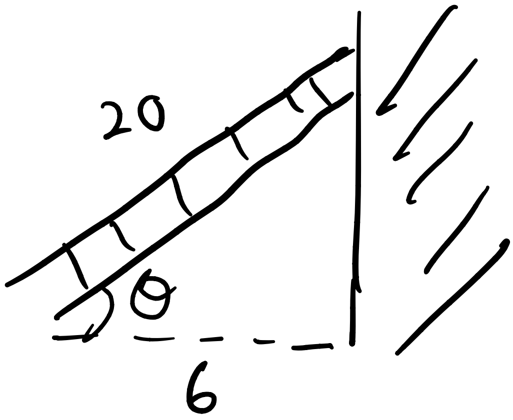
$$\cos\theta = \frac{1}{\sqrt{\tan^2\theta + 1}}$$

$$= \frac{1}{\sqrt{(\tan(\tan^{-1}x))^2 + 1}}$$

$$= \frac{1}{\sqrt{x^2+1}}$$

⑦ Applications

39.



$$\cos\theta = \frac{6}{20}$$

$$\therefore \theta = \cos^{-1}\frac{3}{10}$$

$$= 72.54^\circ$$

$$\therefore h = 20\sin 72.54^\circ$$

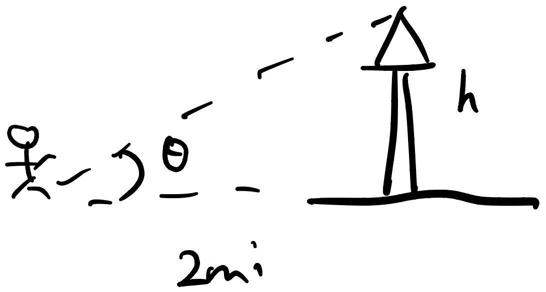
$$= 19.08\text{ ft}$$

40.

$$\tan\theta = \frac{96}{120}$$

$$\theta = 38.66^\circ$$

41.

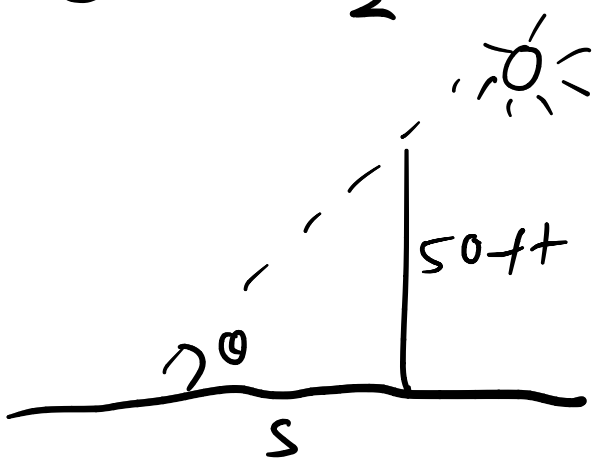


(a) $h = 2\tan\theta$

(b) $\tan\theta = \frac{h}{2}$

$\theta = \tan^{-1}\frac{h}{2}$

42.



(a) $\tan\theta = \frac{50}{s}$

$\theta = \tan^{-1}\frac{50}{s}$

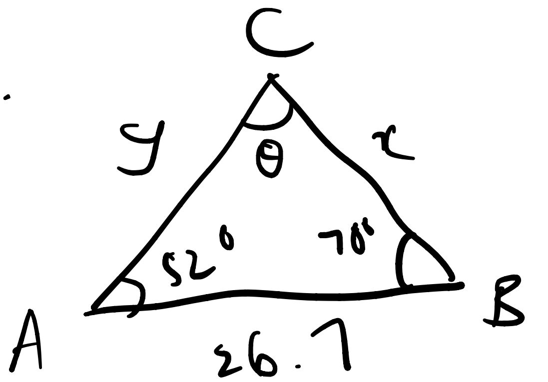
(b) $s = 20$ ft,

$\theta = \tan^{-1}\frac{50}{20}$

$= 68.2$ ft

5.5 The Law of Sines

5.



$$\theta = 180 - 70 - 52^\circ$$
$$= 58^\circ$$

$$\frac{\sin A}{x} = \frac{\sin C}{26.7}$$

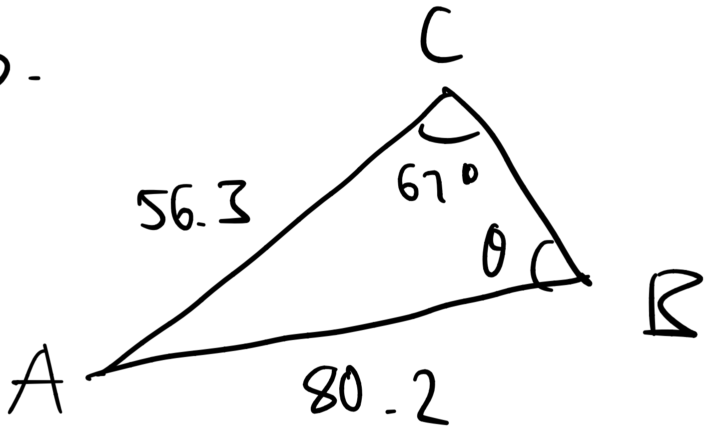
$$\frac{\sin 52^\circ}{x} = \frac{\sin \theta}{26.7}$$

$$\frac{\sin 52^\circ}{x} = \frac{\sin 58^\circ}{26.7}$$

$$x = \sin 52^\circ \Big/ \frac{\sin 58^\circ}{26.7}$$

$$= 24.8$$

6.



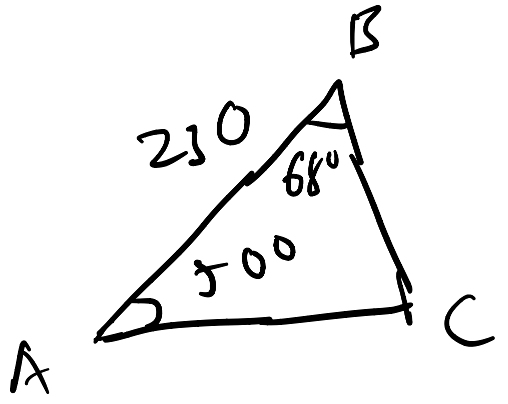
$$\frac{\sin C}{c} = \frac{\sin \theta}{56.3}$$

$$\frac{\sin 67^\circ}{80.2} = \frac{\sin \theta}{56.3}$$

$$\sin\theta = 56.3\left(\frac{\sin 67^\circ}{80.2}\right)$$

$$\theta = 40.3^\circ$$

13. $\angle A = 50°$, $\angle B = 68°$, $c = 230$



$\angle C = 180 - 50 - 68$
$= 62°$

$$\frac{\sin A}{a} = \frac{\sin 62°}{230}$$

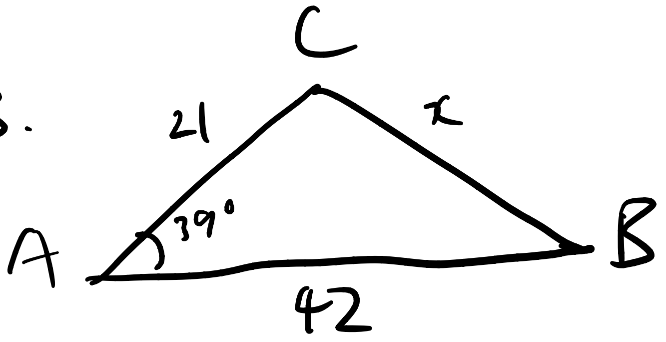
$$a = \frac{230 \sin 50°}{\sin 62°}$$
$= 199.5$

$$\frac{\sin B}{b} = \frac{\sin 62°}{230}$$

$$b = \frac{230 \sin 68°}{\sin 62°}$$
$= 241.5$

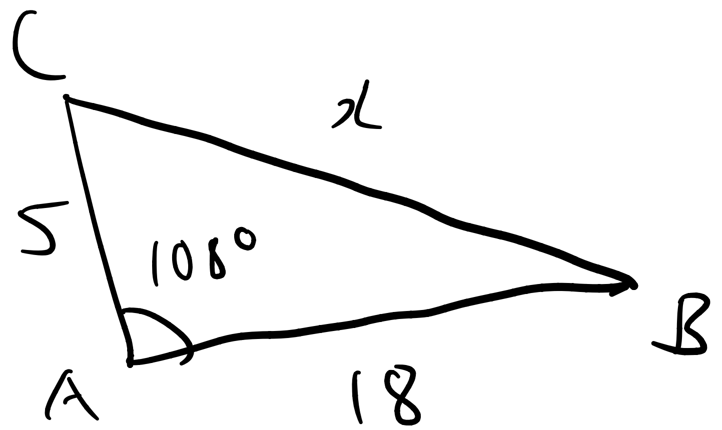
5.6 The Law of Cosines

3.



$$x^2 = 21^2 + 42^2 - 2(21)(42)\cos 39^\circ$$
$$= 834.11$$
$$x = 28.9$$

4.



$$x^2 = 15^2 + 18^2 - 2(15)(18)\cos 108^\circ$$
$$x = 26.8$$

Chapter 9 Vectors in Two and Three Dimensions

8 Examples

Example 1 Describing Vectors in Component Form

(a) $u = \langle 3-(-2), 7-5 \rangle$

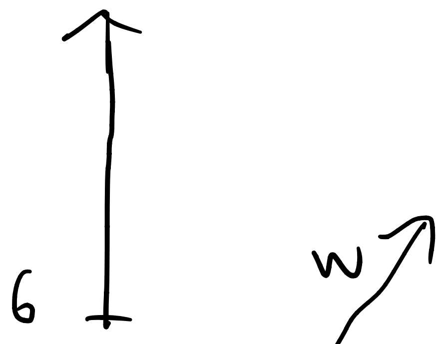
$= \langle 5, 2 \rangle$

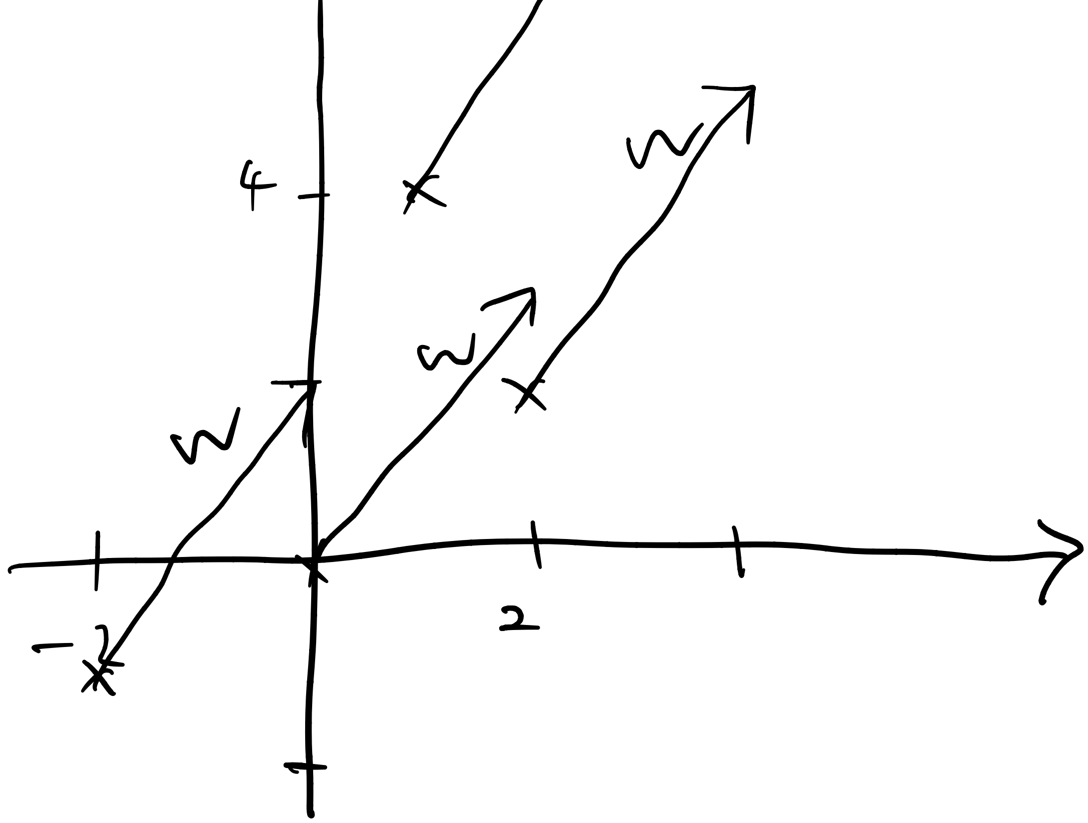
(b) $v = \langle 3, 7 \rangle$, initial point $(2, 4)$

terminal point $(2+3, 4+7)$

$= (5, 11)$

(c) $w = \langle 2, 3 \rangle$



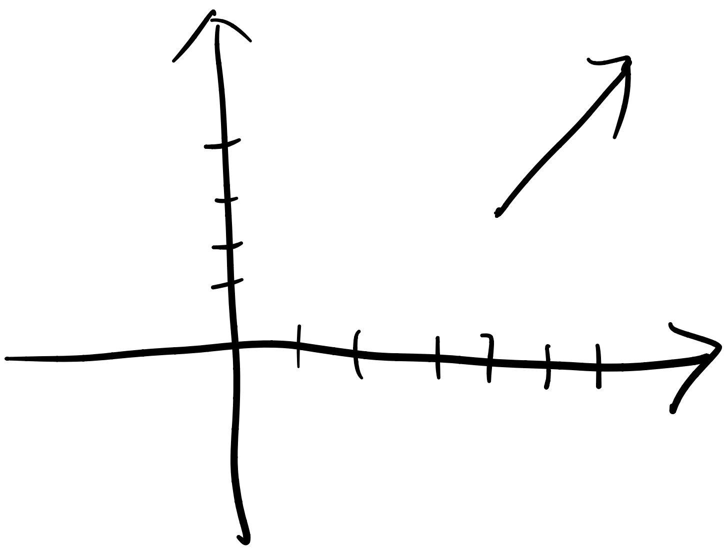

4
w
w
w
w
-2
2

9. 1

11. $P(1,2)$, $Q(4,1)$

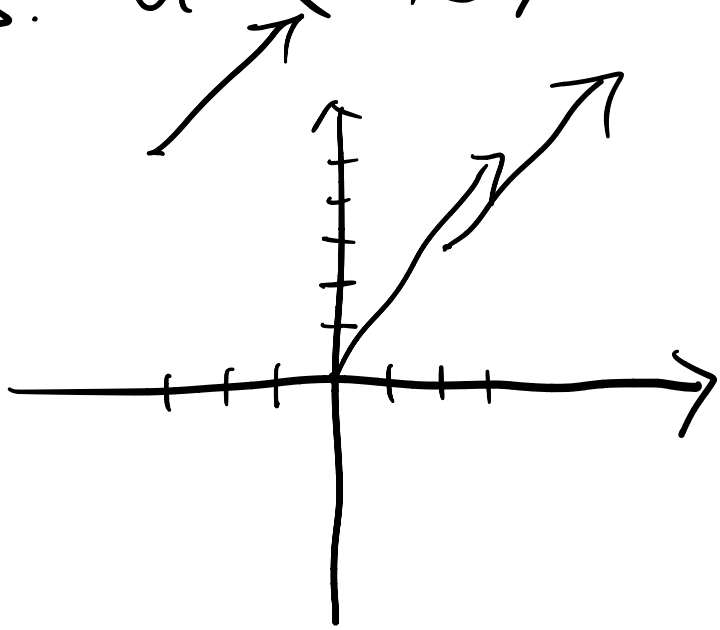
$\vec{V} = \langle 3, 1 \rangle$

19. $u = \langle 2, 4 \rangle$



$v = \langle 6, 7 \rangle$

23. $u = \langle 3, 5 \rangle$



37. $u = 2i + j$, $v = 3i - 2j$ 3/4/2024

$|u| = \sqrt{2^2 + 1^2}$

$= \sqrt{5}$

$|v| = \sqrt{3^2 + (-2)^2}$

$= \sqrt{13}$

$2u = 2(2i + j)$

$= 4i + 2j$

$|2u| = \sqrt{4^2 + 2^2}$

$= 2\sqrt{5}$

$|\frac{1}{2}v| = |\frac{1}{2}|\,|v|$

$= \frac{1}{2}\sqrt{13}$

$= \frac{\sqrt{13}}{2}$

$|u+v|$,

$u + v = (2i + j) + (3i - 2j)$

$= 5i - j$

$|u+v|$

$= \sqrt{5^2 + (-1)^2}$

$= \sqrt{26}$

$|u-v|$,

$u - v = (2i + j) - (3i - 2j)$

$= -i - 3j$

$|u-v|$

$= \sqrt{(-1)^2 + (-3)^2}$

$= \sqrt{10}$

$|u| - |v|$

$= \sqrt{5} - \sqrt{13}$

31. $u = \langle 2, 7 \rangle$, $v = \langle 3, 1 \rangle$

$$2u = \langle 2(2), 2(7) \rangle$$
$$= \langle 4, 14 \rangle$$

$$-3v = \langle -9, -3 \rangle$$

$$u + v = \langle 2+3, 7+1 \rangle$$
$$= \langle 5, 8 \rangle$$

$$3u - 4v = \langle 6, 21 \rangle - \langle 12, 4 \rangle$$
$$= \langle 6-12, 21-4 \rangle$$
$$= \langle -6, 17 \rangle$$

27. $u = \langle 1, 4 \rangle$

$u = i + 4j$

35. $u = 2i$, $v = 3i - 2j$

$$2u = 2(2i)$$
$$= 4i$$

$$-3v = -3(3i - 2j)$$
$$= -9i + 6j$$

$$u + v = 2i + 3i - 2j$$
$$= 5i - 2j$$

$$3u - 4v = 3(2i) - 4(3i - 2j)$$
$$= 6i - 12i + 8j$$
$$= -6i + 8j$$

41. $|v| = 40$, $\theta = 30^\circ$

$v = a_1 i + a_2 j$

$a_1 = |v| \cos\theta$

$= 40 \cos 30$

$= 40\left(\frac{\sqrt{3}}{2}\right)$

$= 20\sqrt{3}$

$a_2 = |v| \sin\theta$

$= 40 \sin 30$

$= 40\left(\frac{1}{2}\right)$

$= 20.$

$v = 20\sqrt{3}\, i + 20\, j$

51. $v = i + \sqrt{3}\, j$

$|v| = \sqrt{1^2 + \sqrt{3}^2}$

$= 2$

$\theta = \tan^{-1} \frac{\sqrt{3}}{1}$

$= 60^\circ$

59. Velocity V,

$V_x = 425$ mi/h

$V_y = 40$ mi/h

(a) $V_y = 40j$

(b) $V_x = 425i$

(c) $V = 425i + 40j$

(d) $|v| = \sqrt{425^2 + 40^2}$

$= 426.88$ mi/h

$\theta = \tan^{-1} \frac{40}{425}$

$= 5.38^\circ$

$\therefore$ Speed is 426.9 mi/h at 5.38° above the x-axis

57. $V_x = -300$ mi/h

$V_y = 30$ mi/h

$$V = -300i + 30j$$

$$\text{Ref } \theta = \tan^{-1} \frac{30}{-300}$$

$$= 5.71^\circ$$

$$\theta = 180^\circ - 5.71$$

$$= 174.29^\circ$$

$\therefore$ The airplane should head in a direction of 185.71° above the x-axis to arrive at a point due west.

67. $F_1 = \langle 2, 5 \rangle$, $F_2 = \langle 3, -8 \rangle$

(a)

$$F_1 + F_2 = \langle 2, 5 \rangle + \langle 3, -8 \rangle$$
$$= \langle 5, -3 \rangle$$

$\therefore \ \Sigma F = \langle 5, -3 \rangle$

(b) If in equilibrium,

$$F_1 + F_2 + F = 0$$

$\therefore \ F = \langle -5, 3 \rangle$

9.2

5. (a)

$u = \langle 2, 0 \rangle, \quad v = \langle 1, 1 \rangle$

$$u \cdot v = a_1 b_1 + a_2 b_2$$
$$= 2 \cdot 1 + 0 \cdot 1$$
$$= 2$$

11. (a) $u = -5j, \quad v = -i - \sqrt{3}j$

$$u \cdot v = 0 \cdot (-1) + (-5) \cdot (-\sqrt{3})$$
$$= 0 + 5\sqrt{3}$$
$$= 5\sqrt{3}$$

5. (b) $u \cdot v = |u||v| \cos\theta$

$$\cos\theta = \frac{u \cdot v}{|u|\,|v|}$$

$|u| = \sqrt{2^2 + 0^2}$
$= 2$

$|v| = \sqrt{1^2 + 1^2}$
$= \sqrt{2}$

$$= \frac{2}{2\sqrt{2}}$$
$$= \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

11. (b) $u \cdot v = |u| \, |v| \cos\theta$

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$|u| = \sqrt{0^2 + (-5)^2} = 5$$

$$= \frac{5\sqrt{3}}{(5)(2)}$$

$$|v| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$$

$$= \frac{\sqrt{3}}{2}$$

15. $u = \langle 6, 4 \rangle$, $v = \langle -2, 3 \rangle$

$$u \cdot v = |u||v| \cos\theta$$

$$|u| = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$$

$$\cos\theta = \frac{u \cdot v}{|u| \, |v|}$$

$$|v| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

$$= \frac{6(-2) + 4(3)}{2\sqrt{13} \cdot \sqrt{13}}$$

$$= 0$$

$\theta = \frac{\pi}{2}$ $\therefore$ They are perpendicular

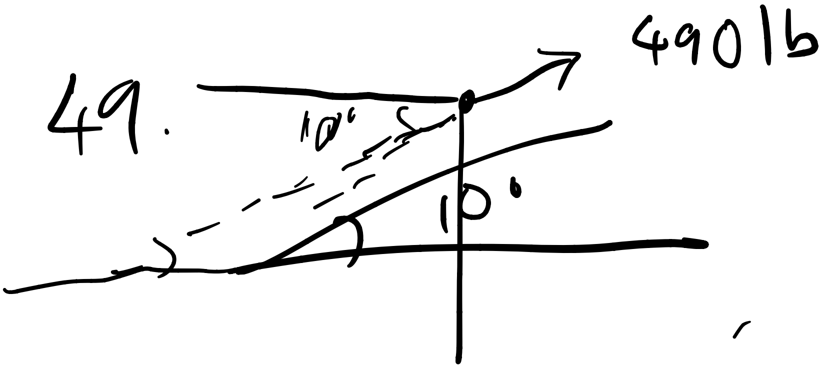
17. $u = \langle -2, -6 \rangle$, $v = \langle 4, 2 \rangle$

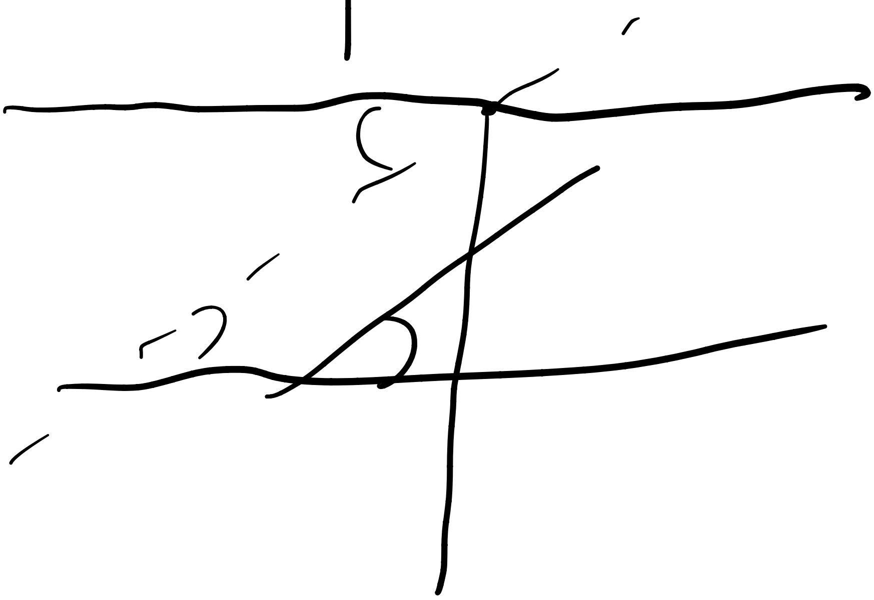
$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$u \cdot v = (-2)(4) + (-6)(2)$$
$$= -8 - 12$$
$$= -20$$

$\therefore$ not perpendicular

49.





$|u| = 490 \text{ lb}$

$W \cos\theta = |u|$

$W = \dfrac{|u|}{\cos\theta}$

$= \dfrac{490}{\cos 80}$

$= 2821.80 \text{ lb}$

$W \sin\theta = F$

$2821.80 \sin 80^\circ = 2778.93 \text{ lb}$

25. $u = \langle 4, 6 \rangle, \ v = \langle 3, -4 \rangle$

$|u| \cos\theta = \dfrac{u \cdot v}{|v|}$

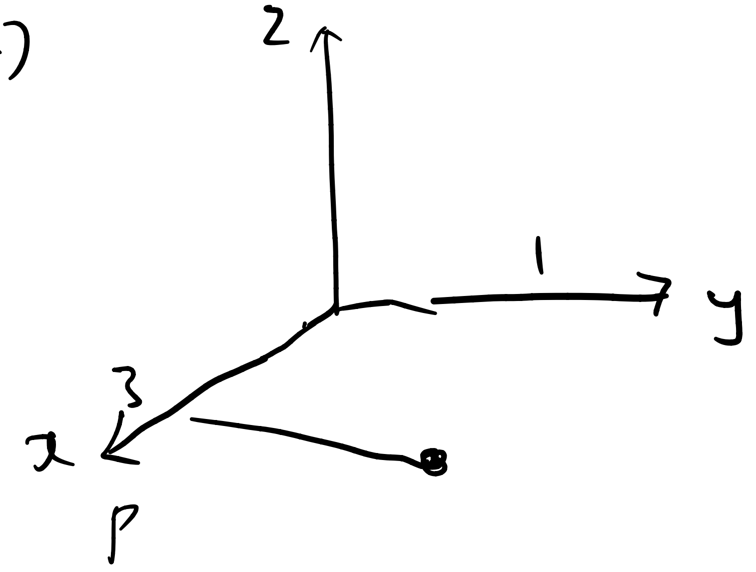
$= \dfrac{4(3) + (6)(-4)}{\sqrt{3^2 + (-4)^2}}$

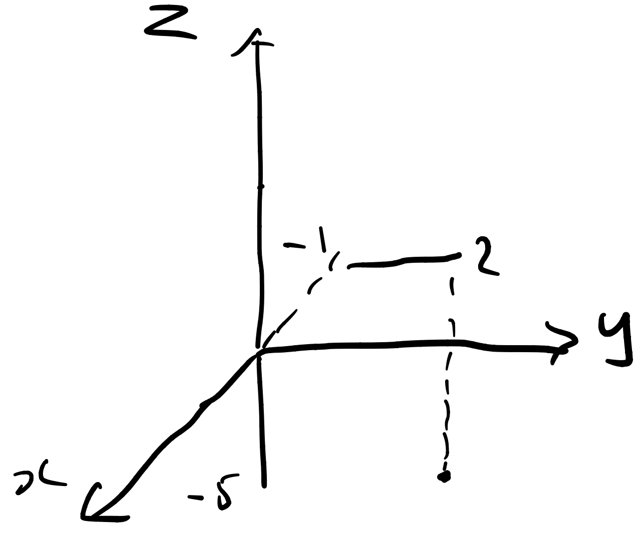
$= \dfrac{12 - 24}{5} = -\dfrac{12}{5}$

9.3

3. $P(3,1,0)$, $Q(-1,2,-5)$

(a)





(b) $d(P,Q)$

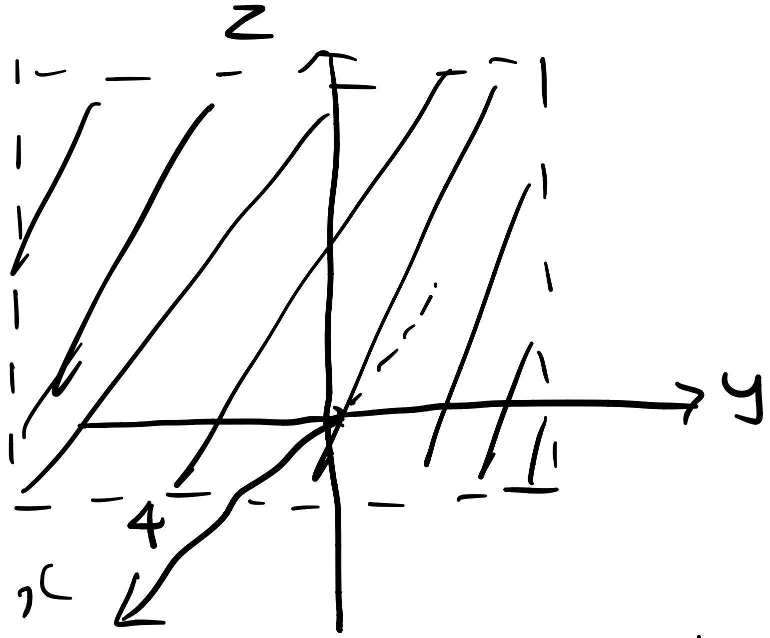
$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$= \sqrt{(3-(-1))^2+(1-2)^2+(0-(-5))^2}$$

$$= \sqrt{4^2+(-1)^2+5^2}$$

$$= \sqrt{42}$$

7. $x = 4$



parallel to yz-plane and 4 units in front of it

11. $r = 5$; $C(2, -5, 3)$

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r^2$$

$$(x - 2)^2 + (y - (-5))^2 + (z - 3)^2 = 5^2$$

$$\therefore (x - 2)^2 + (y + 5)^2 + (z - 3)^2 = 25$$

15. $x^2 + y^2 + z^2 - 10x + 2y + 8z = 9$

$$x^2 - 10x + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2 + y^2 + 2y + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2$$
$$+ z^2 + 8z + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 = 9$$

$$(x - 5)^2 + (y + 1)^2 + (z + 4)^2 = 9 + 25 + 1 + 16$$

$$(x - 5)^2 + (y + 1)^2 + (z + 4)^2 = \sqrt{51}^2$$

$\therefore$ Center of $(5, -1, -4)$ with radius $\sqrt{51}$

19. $(x+1)^2+(y-2)^2+(z+10)^2=10^2$

(a) yz plane

$x=0$,

$(0+1)^2+(y-2)^2+(z+10)^2=100$

$(y-2)^2+(z+10)^2=99$

$(y-2)^2+(z+10)^2=(3\sqrt{11})^2$

thrnce is a circle in the yz plane of center $(0,2,-10)$ and radius $3\sqrt{11}$

(b) $(4+1)^2+(y-2)^2+(z+10)^2=100$

$(y-2)^2+(z+10)^2=(5\sqrt{3})^2$

$\therefore$ a circle in the yz plane of center $(4,2,-10)$ with radius $5\sqrt{3}$

9.4

3. $P(1,-1,0)$, $Q(0,-2,5)$

$$v = \langle 0-1, -2-(-1), 5-0 \rangle$$
$$= \langle -1, -1, 5 \rangle$$

7. $v = \langle 3, 4, -2 \rangle$, $P(2,0,1)$

$$Q = (2+3, 0+4, 1+(-2))$$
$$= (5, 4, -1)$$

11. $v = \langle -2, 1, 2 \rangle$

$$|v| = \sqrt{(-2)^2 + (1)^2 + (2)^2}$$
$$= 3$$

15. $u = \langle 2, -7, 3 \rangle$, $v = \langle 0, 4, -1 \rangle$

$$u + v = \langle 2+0, -7+4, 3+(-1) \rangle$$
$$= \langle 2, -3, 2 \rangle$$

$u-v = \langle 2-0, -7-4, 3-(-1) \rangle$

$= \langle 2, -11, 4 \rangle$

$3u - \frac{1}{2}v = 3\langle 2, -7, 3 \rangle - \frac{1}{2}\langle 0, 4, -1 \rangle$

$= \langle 6, -21, 9 \rangle - \langle 0, 2 - \frac{1}{2} \rangle$

$= \langle 6, -23, 9\frac{1}{2} \rangle$

19. $\langle 12, 0, 2 \rangle$

$v = 12i + 2k$

23. $u = \langle 0, -2, 1 \rangle$, $v = \langle 1, -1, 0 \rangle$

(a) $-2u+3v = \langle -2(0)+3(1), -2(-2)+3(-1), -2(1)+3(0) \rangle$

$= \langle 3, 1, -2 \rangle$

(b) $-2u+3v = 3i + j - 2k$

25. $u = \langle 2, 5, 0 \rangle$, $v = \langle \frac{1}{2}, -1, 10 \rangle$

$$u \cdot v = 2\left(\frac{1}{2}\right) + 5(-1) + 0(10)$$
$$= 1 - 5 + 0$$
$$= -4$$

27. $u = 6i - 4j - 2k$, $v = \frac{5}{6}i + \frac{3}{2}j - k$

$$u \cdot v = 6\left(\frac{5}{6}\right) + (-4)\left(\frac{3}{2}\right) + (-2)(-1)$$
$$= 5 - 6 + 2$$
$$= 1$$

29. $\langle 4, -2, -4 \rangle$, $\langle 1, -2, 2 \rangle$

$$u \cdot v = 4 + 4 - 8$$
$$= 0$$

$\therefore$ yes

37. $3i+4j+5k$ $\quad |v| = \sqrt{3^2+4^2+5^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$\cos\alpha = \dfrac{a_1}{|v|}$

$= \dfrac{3}{5\sqrt{2}}$

$= \dfrac{3\sqrt{2}}{10}$

$\cos\gamma = \dfrac{a_3}{|v|}$

$= \dfrac{5}{5\sqrt{2}}$

$= \dfrac{\sqrt{2}}{2}$

$\cos\beta = \dfrac{a_2}{|v|}$

$= \dfrac{4}{5\sqrt{2}}$

$= \dfrac{2\sqrt{2}}{5}$

$\alpha = \cos^{-1}\left(\frac{3}{10}\sqrt{2}\right)$

$= 64.9°$

$\beta = 55.6°$

$\gamma = 45°$

41. $\alpha = \frac{\pi}{3}$, $\gamma = \frac{2}{3}\pi$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\frac{\pi}{3} + \cos^2\beta + \cos^2\frac{2}{3}\pi = 1$

$\cos^2\beta = 1 - \frac{1}{4} - \frac{1}{4}$

$= \frac{1}{2}$

$\cos\beta = \pm\frac{1}{\sqrt{2}}$

$\beta = \frac{\pi}{4}$

9.5

3. $u = \langle 1, 0, -3 \rangle, \quad v = \langle 2, 3, 0 \rangle$

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{vmatrix}$$

$$= (-1)^2 i \begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} + (-1)^3 j \begin{vmatrix} 1 & -3 \\ 2 & 0 \end{vmatrix}$$

$$+ (-1)^4 k \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix}$$

$$= 9i - 6j + 3k$$

9. $u = \langle 1, 1, -1 \rangle, \quad v = \langle -1, 1, -1 \rangle$

(a) $$u \times v = i \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} - j \begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix}$$

$$+ k \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= 2j + 2k$$

$$|u \times v| = \sqrt{0^2 + 2^2 + 2^2} = 2\sqrt{2}$$

(b) $$\frac{u \times v}{|u \times v|} = \frac{1}{2\sqrt{2}} (2j + 2k)$$

17. $P(0,1,0)$, $Q(1,2,-1)$, $R(-2,1,0)$

$$\vec{PQ} = \langle 1-0, 2-1, -1-0 \rangle$$
$$= \langle 1, 1, -1 \rangle$$

$$\vec{PR} = \langle -2-0, 1-1, 0-0 \rangle$$
$$= \langle -2, 0, 0 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -2 & 0 & 0 \end{vmatrix}$$

$$= i \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} - j \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} + k \begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix}$$

$$= 2j + 2k$$

$$\therefore \; v = \langle 0, 2, 2 \rangle$$

21. $u = \langle 3, 2, 1 \rangle$, $v = \langle 1, 2, 3 \rangle$

$|u \times v|$

$= |u|\,|v| \sin\theta$

$= \sqrt{14}\sqrt{14} \sin 44.4^\circ$

$= 9.80$

$\cos\theta = \dfrac{u \cdot v}{|u|\,|v|}$

$= \dfrac{3+4+3}{\sqrt{14}\sqrt{14}}$

$= \dfrac{10}{14} = \dfrac{5}{7}$

$|u| = \sqrt{3^2+2^2+1^2}$

$= \sqrt{14}$

$|v| = \sqrt{1^2+2^2+3^2}$

$= \sqrt{14}$

$\theta = \cos^{-1}\dfrac{5}{7}$

$= 44.4^\circ$

$u \times v$

$= i \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} - j \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}$

$+ k \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix}$

$= 4i - 8j + 4k$

$u \times v = \langle 4, -8, 4 \rangle$

$|u \times v| = \sqrt{4^2 + (-8)^2 + 4^2}$

$= \sqrt{96}$

$= 4\sqrt{6}$

25. $P(1,0,1)$, $Q(0,1,0)$, $R(2,3,4)$

$$\vec{PQ} = \langle 0-1, 1-0, 0-1 \rangle$$

$$= \langle -1, 1, -1 \rangle$$

$$\vec{PR} = \langle 2-1, 3-0, 4-1 \rangle$$

$$= \langle 1, 3, 3 \rangle$$

$$\triangle PQR = \frac{1}{2} b h$$

$$= \frac{1}{2} |\vec{PQ}| \, |\vec{PR}| \sin\theta$$

$$= \frac{1}{2} |\vec{PQ} \times \vec{PR}|$$

$$= \frac{1}{2} \left\| \begin{vmatrix} i & j & k \\ -1 & 1 & -1 \\ 1 & 3 & 3 \end{vmatrix} \right\|$$

$$= \frac{1}{2} \left| \left(i \begin{vmatrix} 1 & -1 \\ 3 & 3 \end{vmatrix} - j \begin{vmatrix} -1 & -1 \\ 1 & 3 \end{vmatrix} + k \begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} \right) \right|$$

$$= \frac{1}{2} |6i + 2j - 4k|$$

$$= \frac{1}{2} \sqrt{6^2 + 2^2 + (-4)^2}$$

$$= \frac{1}{2} \sqrt{56}$$

$$= \sqrt{14}$$

29. $u = \langle 1, 2, 3 \rangle$, $v = \langle -3, 2, 1 \rangle$,

$w = \langle 0, 8, 10 \rangle$

(a) $v \times w = i \begin{vmatrix} 2 & 1 \\ 8 & 10 \end{vmatrix} - j \begin{vmatrix} -3 & 1 \\ 0 & 10 \end{vmatrix}$

$+ k \begin{vmatrix} -3 & 2 \\ 0 & 8 \end{vmatrix}$

$= 12i + 30j - 24k$

$u \cdot (v \times w)$

$= 12 + 60 - 72$

$= 0$

(b) $\therefore$ vectors u and $v \times w$ are coplanar

9.6

3. $P(1,0,-2)$, $v=\langle 3,2,-3\rangle$

$r=r_0+tv$

$r=\langle x,y,z\rangle$

$=\langle 1+3t, 2t, -2-3t\rangle$

$x=x_0+x_1t$

$=1+3t$

$y=y_0+y_1t$

$=0+2t$

$=2t$

$z=z_0+z_1t$

$=-2-3t$

9. $P(1, -3, 2)$, $Q(2, 1, -1)$

$\overrightarrow{PQ} = \langle 2-1, 1-(-3), -1-2 \rangle$

$= \langle 1, 4, -3 \rangle$

$x = x_0 + at$

$= 1 + t$

$y = y_0 + bt$

$= -3 + 4t$

$z = z_0 + ct$

$= 2 - 3t$

15. $n = \langle 1, 1, -1 \rangle$, $P(0, 2, -3)$

(a) $P_0 = \langle 0, 2, -3 \rangle$

$r = P_0 + vt$

$v = \langle x-0, y-2, z+3 \rangle$
$= \langle x, y-2, z+3 \rangle$

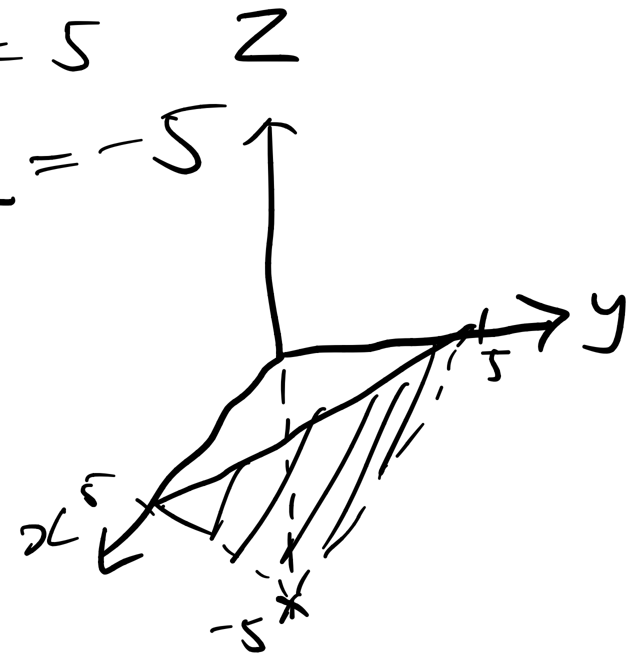
$n \cdot v = 0$

$\langle 1, 1, -1 \rangle \cdot \langle x, y-2, z+3 \rangle = 0$

$x + y - 2 - z - 3 = 0$

$\therefore x + y - z - 5 = 0$

(b) x-intercept: $x = 5$
y-intercept: $y = 5$
z-intercept: $z = -5$



21. $P(6,-2,1)$, $Q(5,-3,-1)$, $R(7,0,0)$

$$\vec{PQ} = \langle 5-6, -3-(-2), -1-1 \rangle$$
$$= \langle -1, -1, -2 \rangle$$

$$\vec{PR} = \langle 7-6, 0-(-2), 0-1 \rangle$$
$$= \langle 1, 2, -1 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -1 & -1 & -2 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= i \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix} - j \begin{vmatrix} -1 & -2 \\ 1 & -1 \end{vmatrix}$$
$$+ k \begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix}$$

$$= i(5) - j(3) + k(-1)$$
$$= 5i - 3j - k$$

$n = \langle 5, -3, -1 \rangle$, $P(6, -2, 1)$

$$r = P_0 + vt$$

$$r - P_0 = \langle x-6, y+2, z-1 \rangle$$

$$n \cdot r = 0$$

$$\langle 5, -3, -1 \rangle \cdot \langle x-6, y+2, z-1 \rangle = 0$$

$$5(x-6) + (-3)(y+2) + (-1)(z-1) = 0$$

$$5x - 30 - 3y - 6 - z + 1 = 0$$

$$\therefore 5x - 3y - z - 35 = 0$$

$$\overrightarrow{QR} = \langle 7-5, 3, 1 \rangle = \langle 2, 3, 1 \rangle$$

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} i & j & k \\ -1 & -1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= i(-1+6) - j(-1+4) + k(-3+2)$$

$$= 5i - 3j - k$$

$\therefore$ $\overrightarrow{QR}$ interchangeable with $\overrightarrow{PR}$ for cross product